INVESTIGATION 20: DOUBLING TIME IN EXPONENTIAL GROWTH

CHECKLIST: Calculations for Problem A, Calculations for Problem B, Questions 1-15

Purpose

- Investigate the mathematical concept of exponential growth, applying doubling time as a calculation method
- Explore the impacts of exponential growth in biological and other processes

Introduction

Growing populations of organisms do not follow linear rates of change. One reason populations grow very rapidly is that they have higher birth rates than death rates. Each cycle of reproduction has more offspring than the previous generation. At any point there are more maturing producers than ever before and the increase in the base population accelerates. Mathematically, such growth is call **exponential**. It is the same type of rate as describes compounding interest in a bank account. While the rate is fixed and may be a small percentage, it is continually applied to a growing base, so that the total expands by a greater and greater amount per unit of time.

The time in which a population or money amount *doubles* is a good benchmark by which to grasp and foresee the impact of exponential growth over time. Even the smallest rate of steady growth leads eventually to doubling and redoubling. While exponential growth in one's investments is welcome, when applied to populations, especially human populations, it can have grave implications. Many people do not have a good grasp of exponential rates. The following two exercises will illustrate the powerful effects of exponential growth when it is modeled as a process of doubling, or repeatedly multiplying by two.

Problems and Questions

Problem A

A math major is home for a vacation break and takes a job for thirty days. In negotiating for a salary, she tells her employer that instead of a wage of $20/hr, she would accept one that pays one penny for the first day, then doubles to two cents the next day, four cents the third day, and so on for the month. The employer thinks this is a good deal, and agrees with the employee's suggestion. Assume a work week is five days, and a work day is eight hours long.

Show your work, including intermediate calculations.

1. Is this a good deal for the boss? If so, under what conditions?
2. How is this a good deal for the math major?
3. When does the student break even – that is, on what day has she made as much as she would have earning $20 per hour?
4. What is the total differential in the two payment methods over the 30-day period?
5. Define exponential growth. Explain why it is so powerful.
6. Describe an example of exponential growth in another field, such as science.
7. Explain what factors might put limits on this type of mathematical increase.

Problem B

Under ideal conditions some common bacteria can divide and double their numbers in less than one-half hour. Suppose one spring day at 6 A.M., a few such bacteria fall into a can of strawberry syrup in a broken garbage bag behind a snack bar. These conditions – warmth, moisture and lots of food – are perfect for growth, and the population doubles every 20 minutes. But by 6 P.M. the bacteria are overcrowded and their food is gone.

As you will discover in your calculations, this story about bacteria dramatizes the uncertain state of our natural resources, even in times of perceptible abundance.

Show your work where applicable. Explain any assumptions you make.

8. At what time did the can of syrup become half full?
9. At one point during the day some forward-thinking bacteria get the idea that they are facing a crisis. Their numbers are growing exponentially and they are using up their space and food at an ever-increasing rate. At what time do you think that idea would come? Explain.
10. Why would awareness of the crisis not occur before 5 P.M.? How much food remains at that time? (Imagine hearing the bacteria politicians saying: "Don't worry, we still have ¾ of our resources. We have more food than we have eaten since we got here.")
11. In spite of the rhetoric, a few bacteria search for more food and space. They find three more syrup cans. How much of a time reprieve are the bacteria given by this find? When will the new cans be depleted?
12. Describe three actions you can take as an individual to help us avoid the fate of the bacteria in the first can.
13. To many of us, Earth does not seem crowded. There are vast, undeveloped areas even in the United States. Explain what "part of the can" is left for us, compared to the bacteria.

General questions…

14. Suppose the global human population growth rate is about 1.3% annually. How long does it take for the human population to double?
15. Given your response to question 5 and online research into Earth's natural resources. How far along are we in terms of Earth's carrying capacity for humans? Briefly describe the kinds of factors to consider.